ION AND ELECTRON FLOW AND SOME CRITICAL RADII IN POLYWELL^{tm} SYSTEMS†

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From examination of EKXL code run output and of analytic studies of particle density distributions,[1,2] a picture emerges of the behavior of ions and electrons in flow through simple[1*] Polywell^{tm} systems.

This shows that the region within the momentum–limited core is net positively charged, and always develops a virtual anode. If the ion current, for given electron drive current, is allowed to become sufficiently large, this virtual anode increases in height above the potential well minimum and the core radius broadens accordingly. This broadening follows the simple algorithm

$$r_c = R\left[\frac{E_{ito}}{E_{ia}}\right]^{0.5} \qquad (1)$$

where E_{ito} is the stable transverse ion energy at the outer boundary after core, edge, and mantle collision processes, and E_{ia} is the potential well depth at the center of the core (i.e. electron maximum injection energy minus virtual anode height). This broadening leads to reduced core density, varying as $(1/r_c)$, for given drive conditions.

However, since the ions approaching the core now must "climb" the increasing anode "hill", they will be slowed down, and their radial motion will decrease with energy given up to the central virtual anode potential. Thus, their convergence will be more rapid than (the usual) geometric inverse–square $(1/r^2)$ dependence that characterizes flow throughout most of the device, and the ion density will increase more rapidly than otherwise expected. This increased densification tends to offset the geometric reduction in

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[1*] By "simple" systems is meant potential wells whose shape is describable by a simple power law expression of the form $\langle r \rangle^m$; with m as m = 3 for the Polywell^{tm}/SCIF systems of interest here

density due to increased core radius. The net result is that fusion reaction rates vary only slowly with increasing anode height up to about half the well depth. Beyond this point the reaction rates steadily decrease due to decreasing fusion reaction cross–sections with the decreasing ion kinetic energy. This feature of ion flow is discussed in more detail in a forthcoming EMC2 Technical Note.[3]

In the region outside the core the plasma is near–neutral (though still slightly positive out to the well minimum, at $r = r_w$) and the ion and electron density follow each other very closely (i.e. within < 1E–5 of local density, for all densities of interest), decreasing about as the inverse–square of the increasing radius. The variation from neutrality is such that the net charge density varies approximately linearly with radius, decreasing to zero at r_w. Beyond r_w the system is nearly neutral, but increasingly slightly negative, out to a radius r_f at which the total energy of the average electron is found to be in well potential energy and transverse kinetic motion (i.e. where the average electron has no radial kinetic energy). This has been called the "stagnation radius" for electron flow;[1] in principle, in the absence of ions, the average electron would never reach a radius smaller than r_f. This radius is just the mean convergence radius of electrons with transverse energy spread $\pm E_{eperp}$ and injection energy E_o, and is given by

$$r_f = R\left[\frac{E_{eperp}}{E_o}\right] \qquad (2)$$

For example, if the electrons are given transverse energy at the outer boundary of $E_{eperp} = 0.6\ E_o$, then the value of $\langle r_f \rangle = (r_f/R)$ will be $\langle r_f \rangle = 0.775$. For a diffuse reflection distribution of electrons from the outer boundary region, $E_{eperp} = E_o/3$ and $\langle r_f \rangle = 0.578$.

The gross density of both species continues to drop approximately as the inverse square from r_f out to a radius $r = r_k$, at which both the ion and electron density begin to increase rapidly as the ions lose energy to the potential well and the electrons pile up to form the main body of this potential well. This region from $r_k < r < R$ is dominated by electron motion. Outside r_k the electron density increases much faster with radius than does the ion density, and the system becomes highly net negative thus ensuring the establishment of the deep negative well required for ion acceleration.

Ion motion dominates the flow within $r < r_f$. Between $r_f < r < r_k$ the transition changes from ion–driven flow (at $r < r_f$) to electron–driven flow (at $r > r_k$). The density at r_k is found from the distributions derived by Krall[2] to be

$$n_i(r) = n_R(3/2)\left[\frac{E_o}{e\phi(r_k)}\right]^{0.5}\left[\frac{R}{r_k}\right]^2 \qquad (3)$$

for a simple power–law well with $m = 3$, where the potential distribution is given by $e\phi(r) = e\phi_o[1-(r/R)^3] = \alpha_q E_o[1-(r/R)^3]$, defining the well depth factor $\alpha_q = e\phi_o/E_o$.

This work[2] gave ion density distributions in three regions across the system. These were in the "core" $r < r_c$, the "mantle" $r_c < r < r_k$, and the outer region $r > r_k$. It also showed that the ion density within a very small radial distance in from the "edge" at $r = R$ would have to rise slightly in order to preserve flow and current continuity within the electron–driven potential well. The "true" monotonic potential well thus starts at a radial position r_p slightly less than R; $r_k << r_p \lesssim R$. From this body of work it is possible to obtain the properties of the flow at the several critical radii just discussed.

Consider the "edge" region, and suppose that the potential well shape is a simple (m) power–law here, so that ion and electron energies would be given by

$$E_i(r) = (1/2)m_i v_i^2 = E_o(1 - \langle r \rangle^m) \tag{4a}$$

$$E_e(r) = (1/2)m_e v_e^2 = E_o(\langle r \rangle^m) \tag{4b}$$

where $\langle r \rangle = (r/R)$. The radius $\langle r_m \rangle$ at which the ion density will be a maximum is that at which the potential well has fallen from r = R to a depth equal to the initial spread in energy of the ion input, E_{io}, here assumed to be uniformly distributed from transverse to radial. The convergence radius is determined by this spread as $\langle r_c \rangle = (E_{io}/E_o)^{0.5}$, so that the "edge" radius is given from the ion eq. (4a) as

$$\langle r_m \rangle = \left[\frac{1 - \langle r_c \rangle^2}{m}\right] \tag{5}$$

which is very small for all convergence ratios and power law exponents of interest. For example, for m = 3 and $\langle r_c \rangle$ = 1E–2, eq. (6) gives $\langle r_m \rangle = 1 - 3.3E{-}5$.

Now consider the radius $\langle r_v \rangle$ at which electron and ion velocities would be equal, assuming the validity of the radial flow model used here. This is readily found from eqs. (4) to be simply

$$\langle r_v \rangle = \left[\frac{m_e}{m_i}\right]^{\frac{1}{m}} \tag{6}$$

For ions of m = 3 and mass $2m_p$ (e.g. D) this gives $\langle r_v \rangle = 0.0650$. Of course, in any real system the existence of finite transverse momenta and their conservation will render this particular numerical result invalid, but it is indicative of the nature of the ion and electron

flow in the system.

In considerable contrast is the radius $<r_p>$ at which ion and electron momenta are equal. Again, from eqs. (4) this is found to be

$$<r_p> = 1 - (1/m)\left[\frac{m_e}{m_i}\right] \tag{7}$$

which yields $<r_p> = 1 - 9.07E{-}5$ for D ions. Note that this is very near to the value for the radius of maximum ion density, $<r_m>$ above. Finally, solving eqs. (4) for the radius $<r_e>$ at which ion and electron energies are equal gives

$$<r_e> = 0.794 \tag{8}$$

The critical radius $<r_k>$ at which the ion density reaches a minimum is found by differentiating the equation (given previously[2]) for ion density in the region between $<r_m>$ and $<r_c>$. This yields the result that

$$<r_k> = 0.830 \tag{10}$$

not far different from the value for $<r_e>$ just obtained.

The ion densities expected from this simple model at these critical radii are summarized below in terms of Krall's edge density[2] n_R, and the core density n_c.

ION DENSITY	IN TERMS OF n_R	IN TERMS OF n_c
At core edge	$n_i = (1.5/\langle r_c \rangle)n_R$	$n_i = n_c$
	for actual density distribution in system	
At radius of minimum density $\langle r_k \rangle$, $\langle r_k \rangle = 0.830$	$n_i = (3.327\langle r_c \rangle)n_R$	$n_i = (2.218\langle r_c \rangle^2)n_c$
	for $(1/r^2)$ density distribution at $r \leq r_k$	
	$n_i = (2.180\langle r_c \rangle)n_R$	$n_i = (1.453\langle r_c \rangle^2)n_c$
At radius of maximum density $\langle r_m \rangle$, $\langle r_m \rangle = (1-\langle r_c \rangle^2/3)$	$n_i = (1.31)n_R$	$n_i = (0.873\langle r_c \rangle)n_c$
At system boundary	$n_i = n_R$	$n_i = (0.667\langle r_c \rangle)n_c$

These formulae are of some interest, as they illustrate the general nature of variation of ion density across the device when it is operating with a stable, filled potential well of simple shape. In particular it is interesting to note that the core ion density is higher than the density at injection by a factor of the order of (R/r_c), and higher than the minimum density in the system by $(R/r_c)^2$. The electron density follows the ion density closely for all radii $r < r_k$. Beyond r_k the electron density increases markedly, becoming comparable to the core density at the system boundary.

In actual fact, this simple model becomes progressively more in error as the outer boundary is approached, because the B field – which determines the potential shape – does not follow a simple power law formula. Rather it varies about as

$$B = B_o\left[\frac{2<r>^m}{(1+<r>^{m+2})}\right] \qquad (11)$$

This "rollover" field shape leads to a flat B field gradient at the system edge, which strongly affects the density distribution in this region. The effect is to flatten the electron distribution considerably from its sharply rising character under the simple model used here. This means that the ion density will also not rise as rapidly with increasing outer radius position, and that all calculations whose answers are driven by the high edge densities of the simple model are suspect and probably in error. Within $r < r_k$ both models are fairly close and the distributions given by the simple model here may be taken as reasonable approximations to real effects. It is only in the outermost regions of the system that significant complexities arise; and these will not be resolved short of good experiments that show particle and field distributions.

REFERENCES

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[2] N. A. Krall, "Density Profiles in the Polywell^{tm}/SCIF Device," Krall Associates Technical Report, KA-90-61, December 1990

[3] Robert W. Bussard, "Virtual Anode Height Variation in Polywell^{tm} Systems," Energy/Matter Conversion Corp. Technical Note, EMC2-0991-02, September 1991